Fair and Efficient Allocations Without Obvious Manipulations

Alexandros Psomasa and Paritosh Verma

Reviewer : Hwichang Jeong

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Seoul National University

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Contribution

- Suggest fractionally Pareto efficient, EF1, NOM allocation mechanism that runs in pseudo-polynomial time.

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Setting

- Consider the problem of allocating m items to n agents.
- A fractional allocation $A \in [0,1]^{n\times m}$ is a matrix that A_{ij} is the fraction of the item j that the agent i receives.
- An integral allocation is $A \in \{0,1\}^{n\times m}$.
- $A = (A_1, \cdots, A_n)^T$ where $A_i = (A_{i1}, \cdots, A_{im}) \in [0,1]^m$ denotes the fraction of all items allocated to agent i.

Allocation

- M : set of items, N : set of agents.
- Each agent $i \in N$ has a private valuation function $\mathrm{v}_i(\cdot)$ that outputs the utility that agent i derives from a set of items.
- Utility of agent i for an allocation A is $\mathrm{v}_i(A_i) = \sum_{j \in M} A_{ij} v_{ij}$.

Mechanism

- A mechanism $\mathcal{M}$ uses reported valuations $b = (b_1, ..., b_n)$ from every agent $i \in N$ and outputs a feasible allocation.
- Deterministic mechanism $\mathcal{M}$ is function outputs an integral allocation based on reported valuations $\mathrm{b} = (\mathrm{b}_1, \cdots, \mathrm{b}_n)^T$.

$$\mathcal{M}(b) = (\mathcal{M}_1(b), \cdots, \mathcal{M}_n(b))^T$$

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Efficiency

- An allocation A is fractionally Pareto efficient (or fPO) iff there is no fractional allocation A' such that for all agents $i \in N$,

$$\mathsf{v}_i(A_i^{'}) \geq \mathsf{v}_i(A_i)$$

  and for at least one agent this inequality is strict.
- An allocation A is α-approximately fractionally Pareto efficient (or α -fPO) iff there is no fractional allocation A' such that for all agents $i \in N$,

$$\alpha \mathsf{v}_i(A_i^{'}) \geq \mathsf{v}_i(A_i)$$

  and for at least one agent this inequality is strict.

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Fairness

- An allocation A is envy-free (EF) if for every pair of agent i, $i' \in N$,

$$\mathrm{v}_i(A_i) \geq \mathrm{v}_i(A_{i'})$$

- Achieving envy-freeness is impossible for integral allocations.
- An integral allocation A is envy-free up to one item (EF1) if for every pair of agent i, $i' \in N$, where $A_j \neq \emptyset$,

$$\mathrm{v}_i(A_i) \geq \mathrm{v}_i(A_{i'} \setminus \{g\})$$

  for some item $g \in A_{i'}$.

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Incentive Compatibility

- A mechanism is called incentive compatible if every agents can achieve the best outcome to themselves just by acting to their private valuation.
- A mechanism is not obvious Manipulable(NOM) if every agent $i \in N$ with private valuation v_i, and every possible report b_i of agent i

$$\min_{\mathsf{v}_{-i}} \mathsf{v}_i(\mathcal{M}_i(\mathsf{v}_i, \mathsf{v}_{-i})) \geq \min_{\mathsf{v}_{-i}} \mathsf{v}_i(\mathcal{M}_i(\mathsf{b}_i, \mathsf{v}_{-i}))$$

$$\max_{\mathsf{v}_{-i}} \mathsf{v}_i(\mathcal{M}_i(\mathsf{v}_i, \mathsf{v}_{-i})) \geq \max_{\mathsf{v}_{-i}} \mathsf{v}_i(\mathcal{M}_i(\mathsf{b}_i, \mathsf{v}_{-i}))$$

  where v_{-i} are reports of the other agents.
- Intuitively, if a mechanism is NOM then an agent cannot increase her worst-case utility or her best-case utility by misreporting her valuation.

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Theorem

- Jugal and Aniket suggest a pseudo-polynomial time deterministic allocation that fPO and EF1.

Theorem
There exists a black-box reduction, which preserves fPO, from the problem of designing a NOM and EF1 mechanism to designing an algorithm that computes clean and non-wasteful and EF1 algorithm.

Clean and non-wasteful

- An allocation A is non-wasteful iff for each $i \in N$, $v_{ij} = 0$ for every unallocated item $j \in M \setminus \cup_{k \in N} A_k$
- An allocation A is clean if for each $i \in N$, $\mathrm{v}_i(g) > 0$ for all $g \in A_i$.

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Black box reduction

- For each agent $i \in N$, let D_i be the set of items that have strictly positive reported value for i

$$D_i = \{j \in M | b_{ij} > 0\}$$

- Let $\hat{D}_i = M \setminus \cup_{i' \neq i} D_{i'}$
- Let R_i be the indicator for the event that the subsets $\{D_{i'}\}_{i' \in N} \setminus \{D_i\}$ are pairwise disjoint.

Black box reduction

- Let $\mathcal{M}^*$ be Clean and non-wasteful fPO and EF1 mechanism.
- Suggest mechanism 1 considers sequentially four cases.

Black box reduction

Case I: The sets $\{D_i\}_{i=1}^{n}$ are pairwise disjoint.

- Allocate the D_i to agent i for each agent $i \in \mathrm{N}$.

Black box reduction

Case II: $R_i = 1$ for exactly one agent $i \in N$.

- This can occur if D_i intersects with two or more $D_{i'}$s
- Allocate the $\hat{D}_i$ to agent i, and the $D_{i'}$ to each agent $i^{'} \in N$, for each $i^{'} \neq i$ if it results in an EF1 allocation.
- Otherwise, allocation returned by the $\mathcal{M}^*$ for the given valuation profile.

Black box reduction

Case III: There are exactly two agents $i, i^{'} \in \mathrm{N}$ such that $R_i = R_{i'} = 1$.

- The only way this is possible is if $D_i, D_{i'}$ intersect each other and any other pair of subsets D_k, D_l where $\{k, l\} \neq \{i, i^{'}\}$, are disjoint.
- Mechanism 1 considers whether the set of goods $D_i \cap D_j$ are valued more by agent i or agent j; each of these two subcases are similar to Case II

Black box reduction

Case IV: None of the previous cases holds (equivalently, $R_i = 0$ for all $i \in \mathrm{N}$).

- Allocate returned by the $\mathcal{M}^*$ for the given valuation profile.